OPTIMIZATION OF PLANT LOCATION PROBLEMS USING SIMULATED ANNEALING – A HEURISTIC APPROACH

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Abstract In Operations Research, many exact and heuristic algorithms have been developed to solve location-allocation problems. However, research on transportation-location problems has been limited to the application of only a few exact solution and heuristic algorithms to small size problems, because the computational effort required grows exponentially with the number of sources and destinations. Here, a small problem is considered to be one that can be solved by an exact solution algorithm. The proposed method gives good results for both small and large test problems. This paper describes a hybrid technique, where the Simulated Annealing portion minimizes the total transportation cost by modifying the source locations. Then, for each location proposed by the Simulated Annealing algorithm, the optimal allocations from each source to each destination are found by solving a linear Transportation problem using traditional Linear Programming techniques. The proposed algorithm was compared to exact solution methods for set of small test problems (using 2 to 4 sources and 4 to 8 destinations), where the exact solution methods could be applied. The algorithm was then tested on two large test problems (10 x 10, 12 X 16) that were constructed in such a way that the exact solution was known. In all cases, the proposed algorithm converged to near-optimal solution.

INTRODUCTION

A transportation–location problem, is a generalized version of the Hitchcock transportation problem (Hadley, 1962), where, in addition to finding the amounts to be shipped from a certain number of origins to another number of destinations, it also finds, the optimal locations of these sources with respect to the fixed locations of the destinations. Cooper (1972) formulated the transportation-location problem as a generalization of both the Hitchcock "Transportation problem" and the "Location-Allocation" problem with unlimited source capacities.

An exact algorithm for the Transportation-Location problem has been developed by Cooper (1972). Cooper's exact algorithm was simply to generate all possible basic feasible solutions to the constraint set of the transportation problem, and then use an iterative procedure to find the optimal location for the given set of allocations. Since, the computational effort required to find an exact solution increases exponentially with the size of the problem, the largest problem could be solved by his method was a 4 x 4 problem. The number of basic solutions generated from 4 x 4 can be upto 11440 and *minimum* number of basic feasible solutions for a transportation–location problem is given as: $n! / (n-m+1)!$, where n is the number of sources, and m is the number of destinations. Therefore, although a 4 x 4 problem has a minimum of 24 basic feasible solutions, but actual number basic feasible solutions generated by Cooper's exact algorithm was 467. A 10 x 10 problem has at least 3.6×10^6 basic feasible solutions, and a 15 x 15 problem has at least 1.3×10^{12} basic feasible solutions. Here the definition of a "large" problem to be one that has more than 1.0×10^6 basic feasible solutions, which is surpassed by a 8 x 11 or a 9 x 10 problem. Anything with a larger number of either sources or destinations will be considered to be "large" for the purpose of this paper.

Cooper (1972) in his "Alternating Transportation-Location Heuristic" generated a set of initial source locations, which yield a set of distances between the sources and destinations. He solved the problem by taking distances as cost coefficients and got optimal allocation from each source to each location. Every time he got a new set of source location and repeated until the amount come within some tolerance. Such iteration yields a convergent, monotone, non-increasing sequence of values for the objective function. Though it got no guarantee for converging to the global optimum for its

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multi-modal but lies within 10 percent of the true optimum. Cooper's (1964) location-allocation (require no capacity constraints on the source) algorithm provided moderately successful results but by no means the best of all other heuristics tested for pure location-allocation problems. Cooper's both transportation-location and location-allocation problems were considered the definitive work until recently where modern algorithms like Simulated Annealing and the Genetic Algorithm emerged for solving large, possibly discrete global optimization problems.

Liu (1994) demonstrated use of Simulated Annealing with rectilinear distances to solve a location-allocation as an effective heuristic approach for solving large-scale problems.

Gonzalez-Monroy and Cordoba (2000) compared both the use of Simulated Annealing and the Genetic Algorithm for the optimization of energy supply systems. The results depicts that the Simulated Annealing is more efficient than the Genetic Algorithm as the size of the problem was increased.

This presented work is based upon Liu's (1994) work on Simulated Annealing. The transportation-location problem is one step more difficult than the location allocation problem taking source capacity as constraints. This work is also more difficult than that performed by Liu, since the use of Euclidean distances, rather than rectilinear distances, makes the objective function non-linear. In addition to solving a more difficult problem than has previously been attempted, an improved method for applying the Simulated Annealing algorithm to these types of problems is presented below.

PROBLEM STATEMENT

Although the general transportation – location problem refers simply to "sources" and "destinations," for practical purposes, we will solve a particular type of a transportation – location problem, namely, identifying the optimal location of new power plants to meet the new (or future) energy demands of a certain number of cities. The objective of this problem will be to minimize the total power distribution cost. The power distribution cost is the sum of the products of the power supply cost (per unit amount, per unit distance), the distance between the plant and the city, and the amount of power supplied from the plant to the city, for all plants and all cities. For each city, we will constrain the total amount of power supplied by all plants to be equal to the total demand of that city. And for each plant, we will constrain the total amount of energy supplied by the plant to be less than or equal to the total capacity of the plant.

The mathematical form of the problem can be written as,

$$Min.Cost\,(C) = \sum_{i=1}^{n} \sum_{j=1}^{m} \phi.\delta_{ij}.v_{ij} \qquad \textit{Eq. 1}$$

subject to;

$$\sum_{i=1}^{n} v_{ij} = d_j \qquad \textit{for} \qquad j = 1, m$$

$$\sum_{j=1}^{m} v_{ij} \le c_i \qquad \textit{for} \qquad i = 1, n$$

Where

ϕ	=	*transportation cost per unit amount per unit distance*
δ_{ij}	=	*distance from source i to destination j*
v_{ij}	=	*amount supplied from source i to destination j*
n	=	*number of plants*
m	=	*number of cities*
x_i , y_i	=	*X & Y coordinates of the source i*
x_j , y_j	=	*X & Y coordinates of the destination j*
d_j	=	*demand of the destination*
c_i	=	*source capacity*

Notice that the Euclidean distance term, δ_{ij}, can be calculated using Eqn. 2 below.

$$\delta_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \qquad \textit{Eq. 2}$$

METHOD

The Transportation-location problem for locating power plants and allocating power to cities is divided into two levels. The functions of each level are described below:

Level 1

At Level 1, the Simulated Annealing algorithm is used to minimize the cost function given in Eqn. 1 above, by adjusting the X and Y locations of each plant. The constraints given in Eqn. 1 are ignored at Level 1, and imposed at Level 2, as will be shown below. At Level 1, the algorithm proceeds as follows:

1. The user enters the locations and demands for each city; the desired starting and ending values for the Boltzman Probability; and the number of Simulated Annealing cycles. These values are used to compute a starting temperature and a temperature reduction factor for the Simulated Annealing algorithm.

2. The X and Y locations of all of the plants are randomly perturbed a small amount from the previous set of values. The objective function is calculated again by calling the Level 2 subroutine.

3. Random locations for the plants are assigned. The objective function (Eqn. 1) is evaluated by calling the Level 2 subroutine, which optimally allocates power from the plants to the demand points, and insures that the constraints are satisfied.

4. If the new objective value is lower, the new locations are accepted. Otherwise the Boltzman Probability is calculated, and a random number between 0 and 1 is generated. If the random number is less than the Boltzman Probability, the new locations are accepted; otherwise they are discarded. Steps 3-4 are repeated until the desired numbers of perturbations per temperature have been performed.

5. The Simulated Annealing temperature is lowered using the temperature reduction factor, which results in a reduction in the Boltzman Probability.

6. Steps 3 - 5 are repeated until the desired number of cycles have been performed.

7. The final cost, the final X and Y locations of the plants, and the corresponding allocations of power from each plant to each city, are reported to the user.

Level 2

The level 2 optimization receives the locations of all the plants from Level 1, and solves a linear Transportation type problem using the Simplex algorithm. The objective and constraints are exactly as shown in Eqn. 1 above. However, since the location of both plants and cities are known at this point, the non-linear distance parameter (Eqn. 2) now becomes a constant, which reduces the problem to a simple location problem, in which the allocations of power from the plants to the cities are adjusted to minimize cost. The optimal cost value, and the optimal allocations from plant i to city j, v_{ij}, are passed back up to Level 1.

For all the sample problems below, the Simulated Annealing algorithm was applied as described above, and a result was obtained. The optimal locations for this first Simulated Annealing run was then used as the starting value for a another Simulated Annealing run using a smaller step size. A more sophisticated algorithm could automatically reduce the step size as the temperature is reduced.

RESULTS

The method described above was applied to the sample problems given in Cooper (1972), and the results were compared to the exact solutions reported by Cooper. These results are shown in Table 1.0 below. Since the method described in this paper involves random perturbations, all the small sample problems were solved 10 times each, and the average result is reported below.

Table 1.0 : Results for Sample Problems Presented in Leon Cooper's Paper

Problem Numbers	Size Source x Dest.	Exact Solution	SA Solution	% Difference
1	2 x 4	54.142	54.145	0.0045
2	2 x 7	50.450	50.450	0.0000
3	2 x 7	72.000	72.010	0.0144
4	2 x 7	38.323	38.323	0.0000
5	2 x 7	48.850	48.850	0.0000
6	2 x 7	38.033	38.037	0.0116
7	2 x 7	44.565	44.565	0.0000
8	2 x 7	59.716	59.717	0.0008
9	2 x 7	62.204	62.209	0.0079

A program was written to implement the Cooper's exact solution method, and another series of small test problems was randomly generated and solved using both Cooper's exact solution technique and the method described in this paper, as shown in Table 2.0 below.
Finally, two large problems were carefully designed so that the optimal value was known in advance. Our method was applied to these large problems, and the results were compared to the exact solution, as shown in Table 3.0 and Table 4.0 below.

DISCUSSION OF RESULTS AND CONCLUSIONS

Results obtained from Tables 1.0 and 2.0, on small problems were very close to the exact solutions (within 0.01 %). However, since an exact solution method is available for small problems, the exact solution would be preferable in all cases where it is applicable.

Table 2.0 : Results for 10 Randomly Generated Problems

Problem No.	Source No. X Destination No.	Exact Solution	SA Solution	% Difference
1	2 x 4	54.1424	54.1431	0.00129
2	2 x 5	65.7816	65.7854	0.00575
3	2 x 6	68.2853	68.2867	0.00205
4	2 x 7	44.1433	44.1433	0.00000
5	2 x 8	93.6597	93.6639	0.00442
6	3 x 3	40.0026	40.0033	0.00159
7	3 x 4	40.0002	40.0009	0.00180
8	3 x 5	60.0000	60.0067	0.01120
9	3 x 6	54.1426	54.1426	0.00006
10	4 x 4	10.0000	10.0008	0.00797

Table 3.0 : Result for 1 Large Problems with known Solutions

Problem No.	Source No. X Destination No.	Exact Solution	SA Solution	% Diff.
1	12 X 16	160.000	160.245	0.15

Table 4.0 – Result for 1 Large Problems with known Solutions

Problem No.	Source No. X Destination No.	Starting Value at (0,0) location	Final Solution	% Improved
1	10 X 10	5441.48	1.775	99.97 %

The real benefit of this method comes for large problems, for which an exact solution is not generally known. The result for the first large problem (12 x 16) in Table 3.0 showed an error of about 0.15 %. The problem converged to very nearly the exact solution, with all plants being located in the correct city. The result for the 2nd large problem also converged very near to the exact solution of "Zero". Because of the zero value of the exact solution the percent difference could not be calculated instead a percent improved had been calculated. The small errors were due to the fact that Simulated Annealing is not particularly effective at driving to the very lowest point of a local optimum. Still, these results illustrate the utility of the two-tiered hybrid Simulated Annealing and Linear Programming method for solving large Plant location problems.

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